Electricity

Alternating Current

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First year

Summary

Summary

Alternating current

Definition

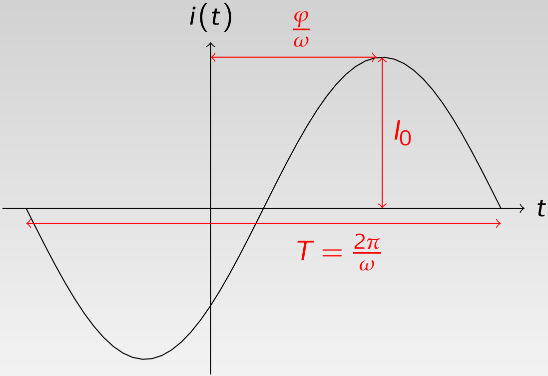
Alternating current (AC) is an electric current which periodically reverses direction, in contrast to direct current (DC) which flows only in one direction.

Most often, the alternating current is sinusoidal :

$$i(t) = I_0 \cos(\omega t + \varphi)$$

- I_0 stands for amplitude
- ω represents *angular frequency* : $\omega = 2\pi \cdot f$
- φ is called *phase*.

Alternating current



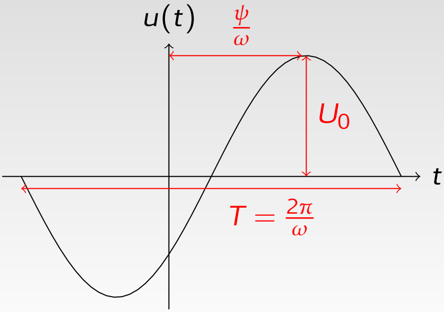
Alternating voltage

Definition

Alternating voltage is a voltage which periodically reverses direction.

Most of the time, it is a sinusoid :

$$u(t) = U_0 \cdot \cos(\omega t + \psi)$$



Some additional definitions

Period : the duration of time of one cycle in a repeating event. Unit : second (s).

Frequency : the number of occurrences of a repeating event per unit of time. Reciprocal of the period. Unit : Hertz (Hz

Root mean square values

Root mean square current

Equal to the direct current that would produce, for the same time, in the same pure resistance, the same amount of heat.

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

Root mean square voltage

Equal to the direct voltage that would produce, for the same time, in the same pure resistance, the same amount of heat.

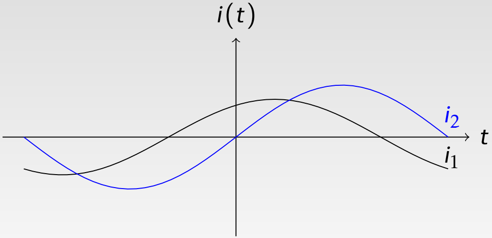
$$U_{rms} = \frac{U_0}{\sqrt{2}}$$

Summation of signals : $\varphi_1 \neq \varphi_2$

$$i_1(t) = I_1 \cdot \cos(\omega t + \varphi_1)$$
$$i_2(t) = I_2 \cdot \cos(\omega t + \varphi_2)$$

The sum is :

$$i_{\text{sum}}(t) = I_1 \cdot \cos(\omega t + \varphi_1) + I_2 \cdot \cos(\omega t + \varphi_2)$$



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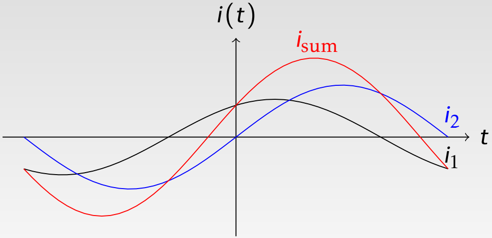


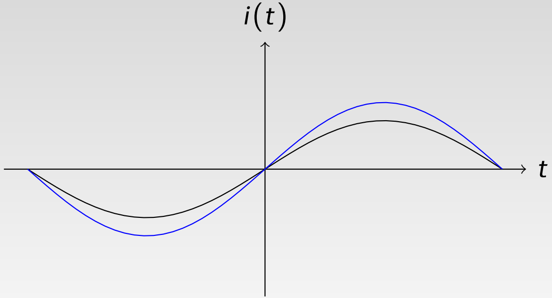
The amplitude of the sum is not equal to the sum of the amplitude.

Summation of signals : $\varphi_1 = \varphi_2$

Signals are in *in phase*

$$i_1(t) = I_1 \cdot \cos(\omega t + \varphi) \qquad i_2(t) = I_2 \cdot \cos(\omega t + \varphi)$$

$$i_{\text{totale}}(t) = I_1 \cdot \cos(\omega t + \varphi) + I_2 \cdot \cos(\omega t + \varphi) = (I_1 + I_2) \cdot \cos(\omega t + \varphi)$$



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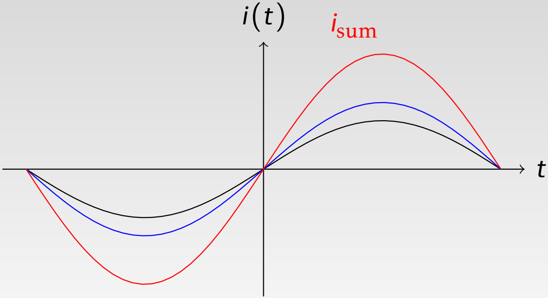


In this only case, the amplitude of the sum is equal to the sum of the amplitude.

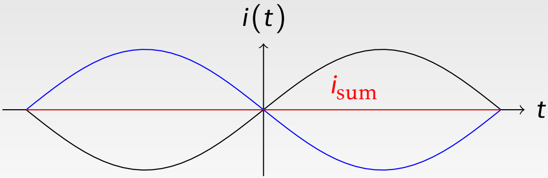
Summation of signals : $\varphi_2 = \varphi_1 + \pi$ and $I_1 = I_2$

Signals are *in phase opposition*

$$i_1(t) = I_0 \cdot \cos(\omega t + \varphi) \qquad i_2(t) = I_0 \cdot \cos(\omega t + \varphi + \pi)$$

But $\cos(a+\pi) = -\cos(a)$, therefore :

$$\begin{aligned} i_{\text{totale}}(t) &= I_0 \cdot \cos(\omega t + \varphi) + I_0 \cdot \cos(\omega t + \varphi + \pi) \\ &= I_0 \cdot \{\cos(\omega t + \varphi) - \cos(\omega t + \varphi)\} \\ &= 0 \end{aligned}$$



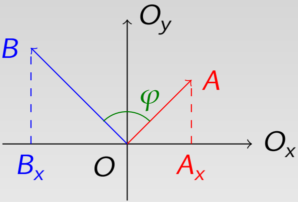
In this case, the amplitude of the sum is zero.

Summary

Fresnel diagramm

Revolving vectors

Two vectors rotating at angular velocity ω.
φ is the phase shift, I_{rms} and U_{rms} their length.



Projection of these vectors on the O_x axis :

- $A_x = I_{rms} \cos(\omega t)$
- $B_x = U_{rms} \cos(\omega t + \varphi)$

Equivalence between alternating voltage/current and these projections.

Fresnel diagramm

Revolving vectors

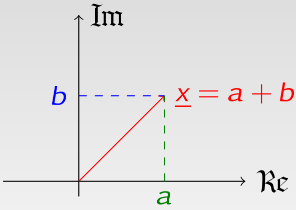
- Adding the vectors adds up the voltages/current
- Very simple for limited operations
- Tedious for more complex configurations
- $\Rightarrow$ Need for a more powerful tool : complex numbers representation

Summary

Cartesian (or algebraic) form

$\underline{x} = a + b\jmath$, where $\jmath^2 = -1$.
a is called *real part* and b *imaginary part*.

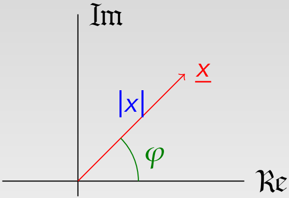
$\underline{x}$ can be viewed as a point of the *complexe plane*, and (a,b) as its *cartesian coordinates*.



Complex numbers can be considered as *position vectors* in a two-dimensionnal plane.

Trigonometric (or polar) form

$\underline{x}$ can be represented through its *absolute value* and *argument* :



From one representation to the other

- From polar to cartesian:
  - $a = |x| \cos \varphi$
  - $b = |x| \sin \varphi$
- From cartesian to polar :
  - $|x| = \sqrt{a^2 + b^2}$
  - $\varphi = atan \frac{b}{a}$

Summary

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Complex current and voltage

- $i(t) = \sqrt{2} I_{rms} \cos(\omega t + \varphi) \Rightarrow \underline{I} = I_{rms} e^{\jmath(\omega t + \varphi)}$
- $u(t) = \sqrt{2} U_{rms} \cos(\omega t + \varphi) \Rightarrow \underline{U} = U_{rms} e^{\jmath(\omega t + \varphi)}$

Every rules seen in the previous chapters are still valid in alternating current, including complex in representation :

- Kirchhoff's laws
- Thevenin's and Norton's theorems
- Superposition theorem
- etc. . .

Why would we use complex representation ?

Because complex is simpler !

- Addition : $\mathfrak{Re}(\underline{A}+\underline{B}) = \mathfrak{Re}(\underline{A}) + \mathfrak{Re}(\underline{B})$ and $\mathfrak{Im}(\underline{A}+\underline{B}) = \mathfrak{Im}(\underline{A}) + \mathfrak{Im}(\underline{B})$
- Multiplication : $|\underline{A}\cdot\underline{B}| = |\underline{A}|\cdot|\underline{B}|$ and $\varphi_{\underline{A}\cdot\underline{B}} = \varphi_{\underline{A}} + \varphi_{\underline{B}}$
- Derivation : $\dfrac{\mathrm{d}\underline{U}}{\mathrm{d}t} = \jmath\omega\underline{U}$
- Integration : $\int \underline{U}\mathrm{d}t = \dfrac{\underline{U}}{\jmath\omega}$

Impedance

Generalised resistance

OHM's law :

$$\underline{U} = \underline{Z}\,\underline{I}$$

$$\underline{Z} = Z e^{j\varphi}$$

Impedance

Generalised resistance

OHM's law :

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$$\underline{Z} = Z e^{\jmath\varphi}$$

- Resistor : $\underline{Z} = R$
- Inductor : $\underline{Z} = \jmath\omega L$
- Capacitor : $\underline{Z} = \dfrac{1}{\jmath\omega C}$

Impedance

Generalised resistance

Ohm's law :

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Except for pure resistors, $\underline{Z}$ is a function of ω.

Summary

Power

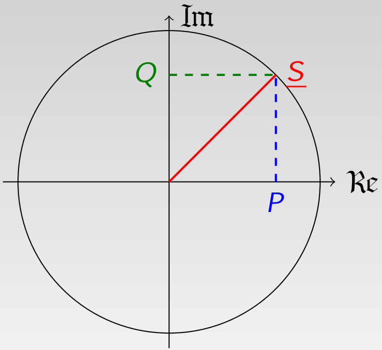
- Power = Energy per unit of time
- DC power : $P = U \cdot I$
- AC power : $p(t) = u(t) \cdot i(t)$

Complex power

One can define four different quantities, all of them homogeneous to power :

- Complex power : $\underline{S} = \underline{U} \cdot \underline{I}$
- Apparent power : $S = |\underline{S}|$
- Active (or real) power : $P = \mathfrak{Re}(\underline{S})$
- Reactive power : $Q = \mathfrak{Im}(\underline{S})$

Complex power



Active power

Definition

Active power P = Average power consumed by the system during a given period of time :

$$P = \frac{1}{T}\int_0^T p(t)\mathrm{d}t = \frac{1}{T}\int_0^T u(t)i(t)\mathrm{d}t$$

If a component has pure real impedance, active power is equal to apparent power.
The unit used for active power is W (watt).

Apparent power

Definition

Apparent power S is the maximum value power can take for given voltage and current.
It is equal to the magnitude of complex power.

Apparent power is noted S. This is also the nominal power specified on electrical equipments.
The unit for apparent power is $\mathrm{V \cdot A}$ (volt-ampere).

Reactive power

Definition

Reactive power Q is the imaginary part of complex power.

A component with pure imaginary impedance dissipates zero active power. In this case, reactive power is equal to apparent power.
The unit for reactive power is var (volt-ampere reactive).

AC power : summary

Apparent, active and reactive power are linked :

$$S^2 = P^2 + Q^2$$

- Active power in W
- Apparent power in $V \cdot A$
- Reactive power in var

W, $V \cdot A$, var are all homogeneous to *power*, but their *physical meaning* is different.

Power factor

Definition

Ratio of active power to apparent power in a circuit is called the power factor.

$$\lambda = \frac{P}{S}$$

This is an intrisic caracteristic of the component.
Calling φ the phase shift between U and I :

$$\lambda = \cos(\varphi)$$

Boucherot's method

Boucherot's theorem

If a circuit contains N components, each of which absorbs active power P_i and reactive power Q_i, then the total active/reactive and powers are the sums of the active/reactive powers of the circuit:

$$P_{tot} = \sum_{i=1}^{N} P_i$$

$$Q_{tot} = \sum_{i=1}^{N} Q_i$$

BOUCHEROT's method

Corollary

Total apparent power *is not* equal to the sum of all apparent powers :

$$S_{tot} \neq \sum_{i=1}^{N} Q_i$$

It can be determined through total active and reactive powers :

$$S_{tot} = \sqrt{P_{tot}^2 + Q_{tot}^2}$$

Summary

In this chapter, we have :

- Described alternative current and voltage
- Defined a formalism based on complex numbers
- Discoverd the concept of impedance
- Distinguished different forms of power