Electricity

Direct Current

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First year

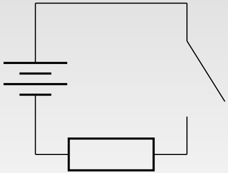
Summary

Summary

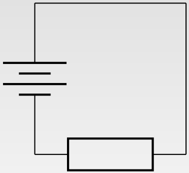
Electrical circuit

By connecting the terminals of a generator to each other by one or more conductive materials, a *closed circuit* is created, in which the electric current can flow.

Open circuit



Closed circuit



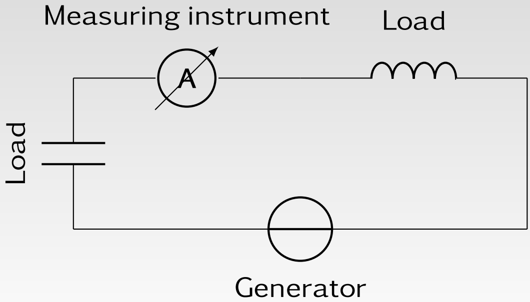
Circuit components

Generators: batteries, current generators, battery packs...

Loads: resistors, inductors, capacitors...

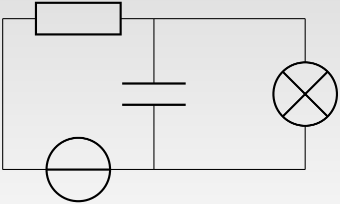
Measuring instruments: voltmeters, ammeters, oscilloscopes...

Safety devices: circuit breakers, fuses...



Nodes and loops

- An electrical network consists of a set of linear dipoles, which are connected by wires of negligible resistance.
- Wires and components form branches, connected together by nodes, and forming loops or meshes.



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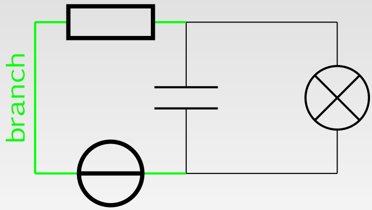

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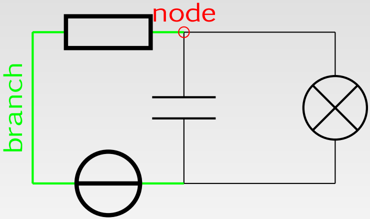

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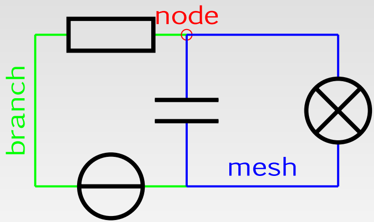
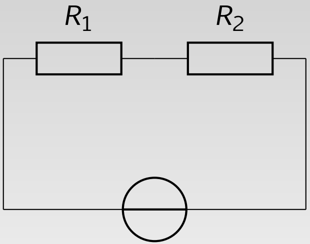


Series connection, equivalent resistance

Components connected in series are connected along a single path, so the same current flows through all of the components.



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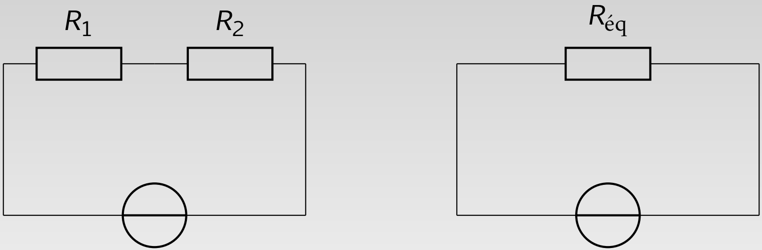
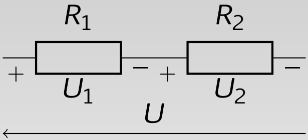


Equivalent resistance is equal to the sum of the resistances.

$$R_{\text{éq}} = R_1 + R_2$$

Series connection : voltage divider

Voltage division is the result of distributing the input voltage among the components of the divider.



The voltage at the terminals of a dipole is :

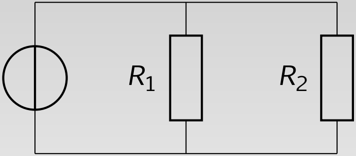
- proportional to the global voltage (U)
- proportionnal to the resistance of the device
- inversely proportionnal to the total resistance of the branch

$$U_1 = U \cdot \frac{R_1}{R_1 + R_2} \qquad U_2 = U \cdot \frac{R_2}{R_1 + R_2}$$

Nota bene : $U_1 + U_2 = U$

Parallel connection : equivalent resistance

Components connected in parallel are connected along multiple paths, so the same voltage is applied to each component.



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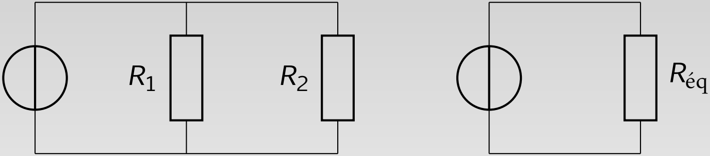


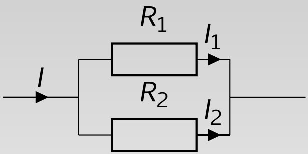
Equivalent conductance is equal to the sum of the conductances :

$$G_{\text{éq}} = G_1 + G_2 \Leftrightarrow \frac{1}{R_{\text{éq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

With $G = \dfrac{1}{R}$

Parallel connection : current divider

Current division is the result of distributing the input current among the components of the divider.



Current through each branch is :

- proportionnal to the total current (I)
- proportionnal to the resistance of *other* branches
- inversely proportionnal to the sum of the resistances in the divider

$$I_1 = I \cdot \frac{R_2}{R_1 + R_2} \qquad I_2 = I \cdot \frac{R_1}{R_1 + R_2}$$

Nota bene $I_1 + I_2 = I$

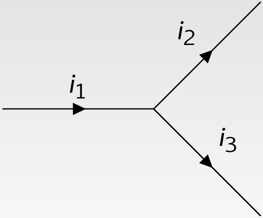
Summary

Kirchhoff's current law

Kirchhoff's nodal rule

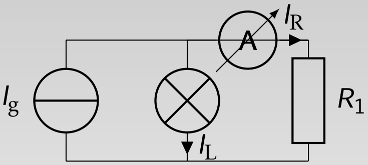
At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node

This law comes from the *conservation of electric charge*



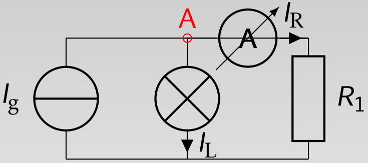
$$i_1 = i_2 + i_3$$

Kirchhoff's nodal rule : example of application



- $I_g = 0{,}5\,A$
- $I_R = 0{,}3\,A$
- How do we find I_L ?

Kirchhoff's nodal rule : example of application



- $I_g = 0{,}5\,A$
- $I_R = 0{,}3\,A$
- How do we find I_L ?

$$I_g = I_R + I_L \Rightarrow I_L = I_g - I_R$$

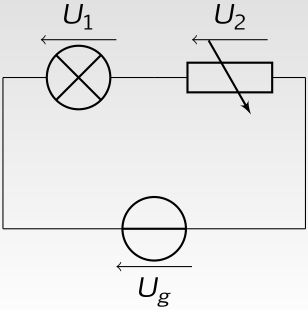
$$I_L = 0{,}2\,A$$

Kirchhoff's voltage law

Kirchhoff's loop rule

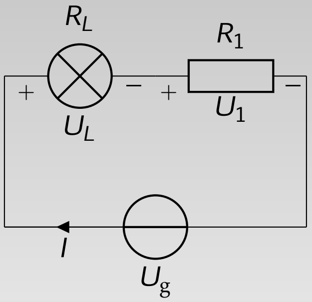
The directed sum of the electrical voltages around any closed network is equal to zero

Comes from the principle of consevation of energy.
Implies that the algebraic sum of the potential drops in a closed loop is equal to the total electromotive forces available in that loop.



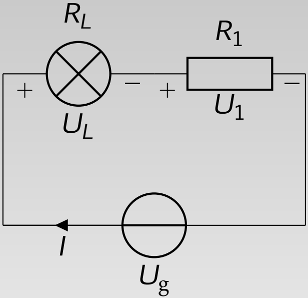
$$U_1 + U_2 = U_g$$

Kirchhoff's loop law : example of application



How do you find the potential drop in the lamp, U_{L}?

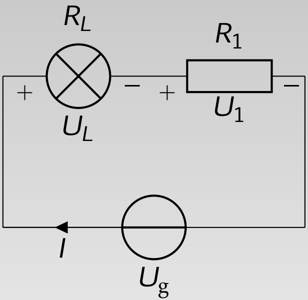
Kirchhoff's loop law : example of application



How do you find the potential drop in the lamp, U_L?

From Ohm's law come $U_1 = R_1 I$ and $U_L = R_L I \Rightarrow U_1 = \dfrac{U_L R_1}{R_L}$

Kirchhoff's loop law : example of application



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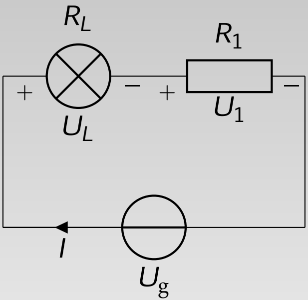
From OHM's law come $U_1 = R_1 I$ and $U_L = R_L I \Rightarrow U_1 = \dfrac{U_L R_1}{R_L}$

There's only one loop in this circuit :

$$U_g = U_L + U_1 = U_L + \frac{U_L R_1}{R_L} = \frac{R_1 + R_L}{R_L} U_L$$

Kirchhoff's loop law : example of application



How do you find the potential drop in the lamp, U_L ?

From OHM's law come $U_1 = R_1 I$ and $U_\mathrm{L} = R_\mathrm{L} I \Rightarrow U_1 = \dfrac{U_\mathrm{L} R_1}{R_\mathrm{L}}$

There's only one loop in this circuit :

$$U_\mathrm{g} = U_\mathrm{L} + U_1 = U_\mathrm{L} + \frac{U_\mathrm{L} R_1}{R_\mathrm{L}} = \frac{R_1 + R_\mathrm{L}}{R_\mathrm{L}} U_\mathrm{L}$$

Finally :

$$U_\mathrm{L} = \frac{R_\mathrm{L}}{R_1 + R_\mathrm{L}} U_\mathrm{g}$$

Summary

Superposition theorem

Superposition theorem

The response (voltage or current) in any branch of a linear circuit having more than one independent source equals the algebraic sum of the responses caused by each independent source acting alone, where all the other independent sources are replaced by their internal impedances.

The statement of this theorem is to be known by heart.

THÉVENIN 's theorem

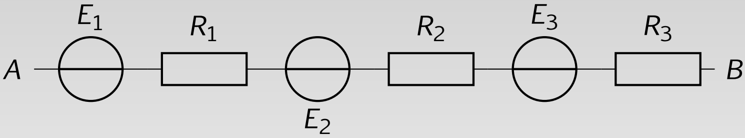
THÉVENIN 's theorem

- Any linear electrical network with voltage and current sources and resistances only can be replaced at terminals A-B by an equivalent voltage source V_{th} in series connection with an equivalent resistance R_{th}.
- The equivalent voltage V_{th} is the voltage obtained at terminals A-B of the network with terminals A-B open circuited.
- The equivalent resistance R_{th} is the resistance that the circuit between terminals A and B would have if all ideal voltage sources in the circuit were replaced by a short circuit and all ideal current sources were replaced by an open circuit.

The statement of this theorem is to be known by heart

Thévenin 's theorem: example of application

Find V_{th} and R_{th} :



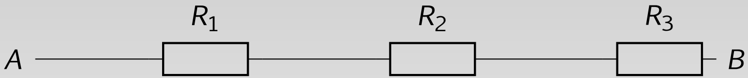
FIrst step Terminals A-B open circuited, there is no current : $I = 0$.
Total voltage is then equal to :

$$E_{\mathrm{Th}} = E_1 - E_2 + E_3$$

THÉVENIN 's theorem: example of application

Second step

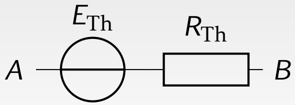
When generators E_1, E_2 et E_3 are replaced by a short circuit, here is the new circuit :



Equivalent resistance of these resistors in parallel connection equals $R_{\mathrm{Th}} = R_1 + R_2 + R_3$.

Finally

The dipole AB is equivalent to this THÉVENIN circuit:



Avec $R_{\mathrm{Th}} = R_1 + R_2 + R_3$ et $E_{\mathrm{Th}} = E_1 - E_2 + E_3$.

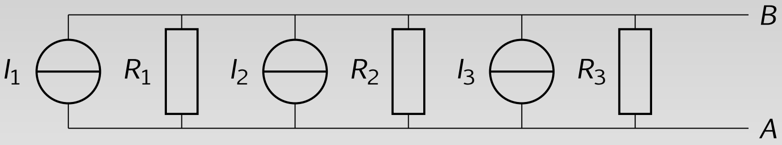
Norton 's theorem

- Any linear electrical network with voltage and current sources and only resistances can be replaced at terminals A–B by an equivalent current source I_N in parallel connection with an equivalent resistance R_N.
- This equivalent current I_N is the current obtained at terminals A-B of the network with terminals A-B short circuited.
- This equivalent resistance R_N is the resistance obtained at terminals A-B of the network with all its voltage sources short circuited and all its current sources open circuited.

The statement of this theorem is to be known by heart.

Norton 's theorem: : example of application

Determine Norton 's current and resistance of the dipole AB:



First step

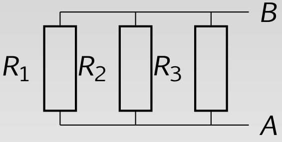
If terminals $A - B$ are short circuited, the voltage U_{AB} is equal to 0.
There is therefore no current in any resistors.
The equivalent current is thus equal to :

$$I_{\mathrm{N}} = I_1 + I_2 + I_3$$

Norton 's theorem : example of application

Second step

When the generators are open circuited, this gives the following graph:

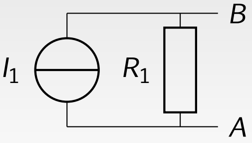


R_N is equivalent to R_1, R_2 et R_3, in parallel connexion :

$$\frac{1}{R_N} = \frac{1}{R_1} + \frac{1}{R_1} + \frac{1}{R_1}$$

Bilan

We finally get Norton's equivalent dipole :



avec $I_N = I_1 + I_2 + I_3$
and $R_N = (R_1^{-1} + R_2^{-1} + R_3^{-1})^{-1}$.

Thévenin -Norton conversion

Thévenin's equivalent and Norton's equivalent are related by the following equations :

From Thévenin to Norton :

$$R_{\mathrm{N}} = R_{\mathrm{Th}}, \quad I_{\mathrm{N}} = E_{\mathrm{Th}}/R_{\mathrm{Th}}$$

From Norton to Thévenin

$$R_{\mathrm{Th}} = R_{\mathrm{N}}, \quad E_{\mathrm{Th}} = I_{\mathrm{N}} R_{\mathrm{N}}$$

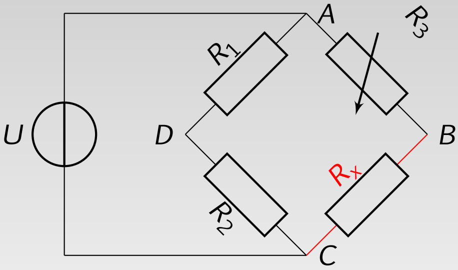
To be noticed

- Equivalent resistance *is not modified*;
- Thévenin 's voltage is related to Norton 's voltage by Ohm's law.

Summary

WHEATSTONE bridge

WHEATSTONE bridge is used to measure unknown electrical resistances.



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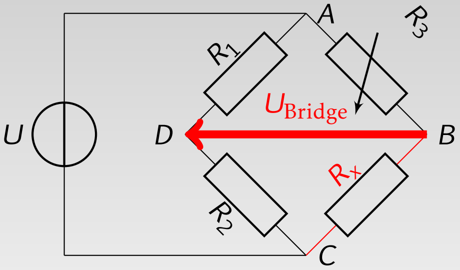


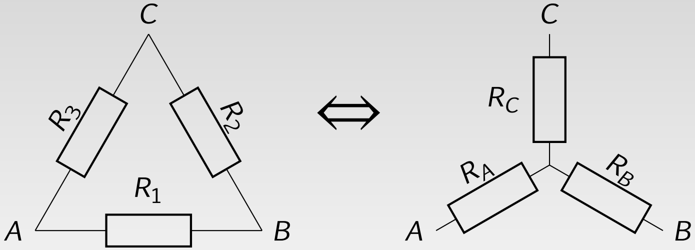
If U_{Pont} is 0, then our bridge is *balanced*. R_x can then be determined:

$$R_x = \frac{R_2 R_3}{R_1}$$

Note: capacities can be measured using SAUTY bridge, and inductances using MAXWELL bridge.

$Y-\Delta$ transform

KENNELLY's theorem establis equivalence between a Y subgraph and a Δ subgraph:



Δ-load to Y-load transformation

$$R_A = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

Y-load to Δ-load transformation

$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

Summary

Wire's resistance

What is the resistance of an electric wire ?

- *Most of the time,* in an electrical circuit, it is negligible compared to the other resistances.
- *In some cases,* however, it cannot be neglected : long range current distribution, high-precision measuring systems, etc.

How resistive is a copper wire of lenght L and cross-section S ?

Basics of dimensional analysis

- In *Physics*, any equations represents *physical quantities* (energy, pressure, current. . .).
- These quantities can be derived from few fundamental quantities.
-

Examples:

- speed ($m \cdot s^{-1}$), is homogeneous to lenght divided by time : $[V] = [L][T]^{-1}$.
- energy (J) is a mass multiplied by a squared speed: $[E] = [M][V]^2 = [M][L]^2[T]^{-2}$.
- voltage (V) is equal to power divided by current: $[V] = [P][A] = [M][L]^2[A]^{-1}[T]^{-3}$.

Resistance of an electric wire: dimensional analysis

An electric wire made of conductive metal. Available data:

- Length L, in m
- Cross-section S, in m^2
- *Resistivity* of te material ρ, in $\Omega \cdot m$

$1\,\Omega \cdot m$ resistivity means a resistance of $1\,\Omega$, for a 1 m long sample, with $1\,m^2$ cross-section.

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$$[R] = ([R][L]) \cdot [L]^{-2} \cdot [L]$$

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$$R = \rho \frac{L}{S}$$

GTE DUNKERQUE

Temperature coefficient

- Resistivity is a function of temperature: $\rho = f(T)$.
- Over a sufficiently narrow temperature range, this function is *linear*.
- Evolution is determined by α, temperature coefficient :

$$\rho = \rho_0(1 + \alpha \Delta T)$$

Conclusion

In this chapter, we have…

- described electrical circuits as dipoles connected together through node, branchs and loops
- studied KIRCHHOFF's laws as well as voltage dividers and current dividers
- learned how to simplify complex circuit graphs with THÉVENIN and NORTON
- discovered the basics of dimensionnal analysis