

Exercice 1.

1. On a

$$\begin{aligned}
 4x^2 - 9 = 0 &\iff (2x - 3)(2x + 3) = 0 \\
 &\iff 2x - 3 = 0 \quad \text{ou} \quad 2x + 3 = 0 \\
 &\iff x \in \left\{ -\frac{3}{2}, \frac{3}{2} \right\}
 \end{aligned}$$

2. On a

$$\begin{aligned}
 \frac{4x^2 - 9}{x - \frac{3}{2}} = 0 &\iff \begin{cases} 4x^2 - 9 = 0 \\ x - \frac{3}{2} \neq 0 \end{cases} \\
 &\iff \begin{cases} x = -\frac{3}{2} \quad \text{ou} \quad x = \frac{3}{2} \\ x \neq \frac{3}{2} \end{cases} \\
 &\iff x = -\frac{3}{2}
 \end{aligned}$$

3. On a

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

4. Un produit de facteur étant nul si et seulement si l'un au moins des facteurs est nul,

$$\begin{aligned}
 t^3 - 6t^2 + 12t - 8 = 0 &\iff (t - 2)^3 = 0 \\
 &\iff t - 2 = 0 \\
 &\iff t = 2
 \end{aligned}$$

5. On a $x^2 - x - 2 = (x + 1)(x - 2)$

6. On a :

$$\begin{aligned}
 \frac{x^2 + 4x}{x - 3} \geq \frac{5x + 2}{x - 3} &\iff \frac{x^2 - x - 2}{x - 3} \geq 0 \\
 &\iff \frac{(x + 1)(x - 2)}{x - 3} \geq 0
 \end{aligned}$$

d'où, grâce au tableau de signes 1 page suivante :

$$\frac{x^2 + 4x}{x - 3} \geq \frac{5x + 2}{x - 3} \iff x \in [-1, 2] \cup [3, +\infty[$$

Exercice 2.

x	$-\infty$	-1	2	3	$+\infty$	
$x + 1$	-	0	+	+	+	
$x - 2$	-	-	0	+	+	
$x - 3$	-	-	-	0	+	
$\frac{(x+1)(x-2)}{x-3}$	-	0	+	0	-	+

TABLE 1 – Tableau de signe de $\frac{(x+1)(x-2)}{x-3}$

x	$-\infty$	-1	1	$+\infty$
$ x + 1 $	$-x - 1$	0	$x + 1$	$x + 1$
$ x - 1 $	$-x + 1$	$-x + 1$	0	$x - 1$
$f(x)$	-2	$2x$	2	

TABLE 2 – Expression de $f(x)$ sans valeur absolue

1. Le tableau 2 permet d'exprimer $f(x)$ sans valeur absolue :

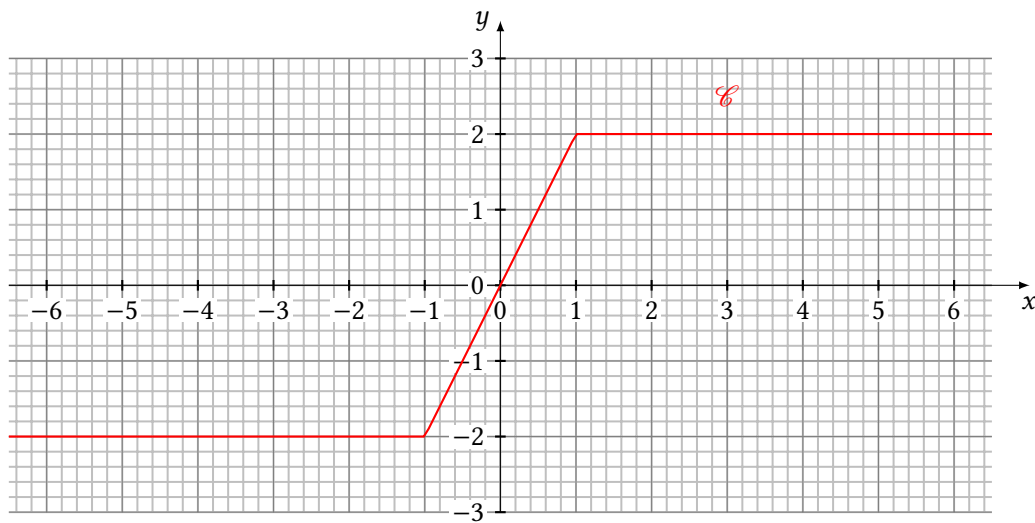
$$f(x) = \begin{cases} -2 & \text{si } x \in]-\infty, -1[\\ 2x & \text{si } x \in [-1, 1] \\ 2 & \text{si } x \in]1, +\infty[\end{cases}$$

2. La courbe \mathcal{C} de la fonction f est représentée à la figure 1 page suivante.

Exercice 3.

1. On a :

$$\begin{aligned} \sin^2 x (1 + \cotan^2 x) &= \sin^2 x \left(1 + \frac{\cos^2 x}{\sin^2 x} \right) \\ &= \sin^2 x + \cos^2 x \\ &= 1 \end{aligned}$$

FIGURE 1 – Courbe de la fonction f

2. Par les formules de l'arc double et fondamentale de la trigonométrie circulaire, on a :

$$\begin{aligned} (2r \sin \theta \cos \theta)^2 + r^2(\cos^2 \theta - \sin^2 \theta)^2 &= r^2(\sin 2\theta)^2 + r^2(\cos 2\theta)^2 \\ &= r^2(\sin^2 2\theta + \cos^2 2\theta) \\ &= r^2 \end{aligned}$$

3. (a) On sait que $\cos 2\alpha = 2 \cos^2 \alpha - 1$.

(b) On sait en outre que $\sin 2\alpha = 2 \sin \alpha \cos \alpha$. Donc

$$\begin{aligned} \cos 3\alpha &= \cos(2\alpha + \alpha) \\ &= \cos 2\alpha \cos \alpha - \sin 2\alpha \sin \alpha \\ &= (2 \cos^2 \alpha - 1) \cos \alpha - 2 \sin \alpha \cos \alpha \sin \alpha \\ &= (2 \cos^2 \alpha - 1) \cos \alpha - 2 \cos \alpha \sin^2 \alpha \\ &= (2 \cos^2 \alpha - 1) \cos \alpha - 2 \cos \alpha (1 - \cos^2 \alpha) \\ &= 4 \cos^3 \alpha - 3 \cos \alpha \end{aligned}$$

4. On a $\frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$ donc

$$\begin{aligned} \cos\left(\frac{\pi}{12}\right) &= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\ &= \cos\left(\frac{\pi}{3}\right) \cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right) \sin\left(\frac{\pi}{4}\right) \\ &= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned}\sin\left(\frac{\pi}{12}\right) &= \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\ &= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

5. On a

$$\begin{aligned}\cos\left(\frac{11\pi}{12}\right) &= \cos\left(\frac{12\pi - \pi}{12}\right) \\ &= \cos\left(\pi - \frac{\pi}{12}\right) \\ &= -\cos\left(\frac{\pi}{12}\right) \\ &= -\frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\sin\left(\frac{11\pi}{12}\right) &= \sin\left(\pi - \frac{\pi}{12}\right) \\ &= \sin\left(\frac{\pi}{12}\right) \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\cos\left(\frac{5\pi}{12}\right) &= \cos\left(\frac{6\pi - \pi}{12}\right) \\ &= \cos\left(\frac{\pi}{2} - \frac{\pi}{12}\right) \\ &= \sin\left(\frac{\pi}{12}\right) \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\sin\left(\frac{5\pi}{12}\right) &= \sin\left(\frac{\pi}{2} - \frac{\pi}{12}\right) \\ &= \cos\left(\frac{\pi}{12}\right) \\ &= \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

Exercice 4.

1. On a :

$$\begin{aligned}\cos \frac{\pi}{4} &= \cos \left(2 \times \frac{\pi}{8} \right) \\ &= 2 \cos^2 \frac{\pi}{8} - 1\end{aligned}$$

Or $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ donc :

$$2 \cos^2 \frac{\pi}{8} - 1 = \frac{\sqrt{2}}{2}$$

si bien que

$$\begin{aligned}\cos^2 \frac{\pi}{8} &= \frac{1 + \frac{\sqrt{2}}{2}}{2} \\ &= \frac{2 + \sqrt{2}}{4}\end{aligned}$$

On en déduit que :

$$\begin{aligned}\left| \cos \frac{\pi}{8} \right| &= \sqrt{\frac{2 + \sqrt{2}}{4}} \\ &= \frac{\sqrt{2 + \sqrt{2}}}{2}\end{aligned}$$

Or $0 \leq \frac{\pi}{8} \leq \frac{\pi}{2}$ donc $\cos \frac{\pi}{8} \geq 0$ si bien que $\left| \cos \frac{\pi}{8} \right| = \cos \frac{\pi}{8}$. Il s'ensuit que

$$\cos \frac{\pi}{8} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

2. (a) On a :

$$\begin{aligned}\sin^2 \frac{\pi}{8} &= 1 - \cos^2 \frac{\pi}{8} \\ &= 1 - \frac{2 + \sqrt{2}}{4} \\ &= \frac{2 - \sqrt{2}}{4}\end{aligned}$$

On en déduit que :

$$\begin{aligned}\left| \sin \frac{\pi}{8} \right| &= \sqrt{\frac{2 - \sqrt{2}}{4}} \\ &= \frac{\sqrt{2 - \sqrt{2}}}{2}\end{aligned}$$

Or $0 \leq \frac{\pi}{8} \leq \frac{\pi}{2}$ donc $\sin \frac{\pi}{8} \geq 0$ si bien que $\left| \sin \frac{\pi}{8} \right| = \sin \frac{\pi}{8}$. Il s'ensuit que

$$\sin \frac{\pi}{8} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

(b) Par définition :

$$\begin{aligned}\tan \frac{\pi}{8} &= \frac{\sin \frac{\pi}{8}}{\cos \frac{\pi}{8}} \\ &= \frac{\frac{\sqrt{2-\sqrt{2}}}{2}}{\frac{\sqrt{2+\sqrt{2}}}{2}} \\ &= \frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}} \\ &= \sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} \\ &= \sqrt{\frac{(2-\sqrt{2})^2}{(2+\sqrt{2})(2-\sqrt{2})}} \\ &= \frac{|2-\sqrt{2}|}{\sqrt{2}} \\ &= \frac{2-\sqrt{2}}{\sqrt{2}} \\ &= \sqrt{2}-1\end{aligned}$$